Exercise 83

The motion of a spring that is subject to a frictional force or a damping force (such as a shock absorber in a car) is often modeled by the product of an exponential function and a sine or cosine function. Suppose the equation of motion of a point on such a spring is

$$s(t) = 2e^{-1.5t}\sin 2\pi t$$

where s is measured in centimeters and t in seconds. Find the velocity after t seconds and graph both the position and velocity functions for $0 \leq t \leq 2$.

Solution

The velocity is the derivative of the displacement function.

$$\begin{aligned}
v(t) &= \frac{ds}{dt} \\
&= \frac{d}{dt}\left(2e^{-1.5t}\sin 2\pi t\right) \\
&= 2\frac{d}{dt}\left(e^{-1.5t}\sin 2\pi t\right) \\
&= 2\left\{\left[\frac{d}{dt}(e^{-1.5t})\right]\sin 2\pi t + e^{-1.5t}\left[\frac{d}{dt}(\sin 2\pi t)\right]\right\} \\
&= 2\left\{\left[(e^{-1.5t})\cdot\frac{d}{dt}(-1.5t)\right]\sin 2\pi t + e^{-1.5t}\left[(\cos 2\pi t)\cdot\frac{d}{dt}(2\pi t)\right]\right\} \\
&= 2\left\{\left[(e^{-1.5t})\cdot(-1.5)\right]\sin 2\pi t + e^{-1.5t}\left[(\cos 2\pi t)\cdot(2\pi)\right]\right\} \\
&= 2e^{-1.5t}(-1.5\sin 2\pi t + 2\pi\cos 2\pi t)
\end{aligned}$$

Since s is in centimeters and t is in seconds, ds/dt is in centimeters per second.